# Solutions Test Drill I Test Drill II 

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## Analog circuit Drill

Q1. Assume linear operation to get


Since, calculation gives $\mathrm{v}_{\text {out }}>15 \mathrm{~V}$,op-amp is in saturation. In fact , $\mathrm{v}_{\text {out }}=15 \mathrm{~V}$ and $\mathrm{v}_{\mathrm{d}}=15 \times 10^{-4} \mathrm{~V}=1.5 \mathrm{mV}$

$$
10 \mathrm{k} \Omega
$$

Q2. $\mathrm{f}=100 \mathrm{~Hz}$

$$
\begin{aligned}
& \mathrm{T}=\frac{1}{100}=0.01=10 \mathrm{msec} \\
& \begin{aligned}
\text { dc component } & =\frac{\text { area over one time period }}{\mathrm{T}}=\frac{\mathrm{A}_{1}+\mathrm{A}_{2}}{\mathrm{~T}} \\
& =\frac{9.4 \times 5 \times 10^{-3}-8.8 \times 5 \times 10^{-3}}{10 \times 10^{-3}}=0.3 \mathrm{~V}
\end{aligned}
\end{aligned}
$$




Q3. Let $\mathrm{v}_{\mathrm{in}}=\mathrm{A} \sin \omega_{0} \mathrm{t}$. Then, $v_{\text {out }}=-\left(10 \times 10^{3} \times 0.01 \times 10^{-6}\right) \frac{\mathrm{d}}{\mathrm{dt}}\left(\mathrm{A} \sin \omega_{0} \mathrm{t}\right)$

$$
=-10^{-4} \mathrm{~A} \omega_{0} \cos \omega_{0} \mathrm{t}
$$

$$
\left|\mathrm{v}_{\text {in }}\right|=\left|\mathrm{v}_{\text {out }}\right| \Rightarrow \mathrm{A}=10^{-4} \mathrm{~A} \omega_{0} \Rightarrow \omega_{0}=10 \mathrm{krad} / \mathrm{sec} \text { and } \mathrm{f}_{0}=\frac{10}{2 \pi}=1.59 \mathrm{kHz}
$$

Q4. (c) $V_{1}-V_{2}=3.2 V$ as demonstrated below.


Test Drill I

Q5. The output resistance

$$
\begin{aligned}
\mathrm{r}_{0}=\frac{\mathrm{V}_{\mathrm{A}}}{\beta \mathrm{I}_{\mathrm{B}}} & =\frac{200}{100 \times 50 \times 10^{-6}}=4 \times 10^{4} \Omega \\
& =40 \mathrm{k} \Omega
\end{aligned}
$$



Q6. (c) When $V_{x}$ varies sinusoidally from zero to peak value 10 V , the capacitor $\mathrm{C}_{1}$ charges instantaneously to 10 V with polarity as shown, while $D_{l}$ remains on. After $\mathrm{C}_{1}$ has charged to $10 \mathrm{~V}, D_{I}$ turns off and in steady state, $\mathrm{V}_{1}=\mathrm{V}_{\mathrm{x}}-10=10 \cos \omega \mathrm{t}-10$ has extremities at 0 and -20 V as depicted below. While $\mathrm{V}_{1}$ varies from 0 to $-20 \mathrm{~V}, \mathrm{D}_{2}$ turns on to allow the capacitor $\mathrm{C}_{2}$ to charge instantaneously to 20 V with polarity as shown . Thereafter, $\mathrm{D}_{2}$ turns off and $\mathrm{C}_{2}$ does not find any path to discharge. Thus, $\mathrm{V}_{\mathrm{Y}}=-20 \mathrm{~V}$.



Q7.(d) $\alpha=1 \Rightarrow \mathrm{I}_{\mathrm{B}}=0$ and $\mathrm{I}_{\mathrm{C}}=\mathrm{I}_{\mathrm{E}}=1 \mathrm{~mA}$
$\frac{-4-(-10)}{\mathrm{R}_{\mathrm{C}}}=\frac{10-0.7}{\mathrm{R}_{\mathrm{E}}}=1 \mathrm{~mA}$
$R_{E}=9.3 \mathrm{k} \Omega$ and $R_{C}=6 \mathrm{k} \Omega$


Q8(b). $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ both are in saturation.
$\mathrm{K}\left(\mathrm{V}_{\mathrm{GS} 1}-\mathrm{V}_{\mathrm{t}}\right)^{2}=\mathrm{K}\left(\mathrm{V}_{\mathrm{GS} 2}-\mathrm{V}_{\mathrm{t}}\right)^{2} ; \mathrm{V}_{\mathrm{GS} 1}=\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{B}}$ and $\mathrm{V}_{\mathrm{GS} 2}=\mathrm{V}_{\mathrm{A}}$
$\left(V_{D D}-V_{B}-V_{t}\right)^{2}=\left(V_{A}-V_{t}\right)^{2}$
$V_{D D}-V_{B}-V_{t}= \pm\left(V_{A}-V_{t}\right)$
either $V_{D D}-V_{B}-V_{t}=V_{A}-V_{t} \Rightarrow V_{D D}=V_{A}+V_{B}$
or $\quad V_{D D}-V_{B}-V_{t}=V_{t}-V_{A}$
but $\mathrm{V}_{\mathrm{GS}}=\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{B}}$ must be greater than $\mathrm{V}_{\mathrm{t}}$, that is, $\mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{B}}-\mathrm{V}_{\mathrm{t}}$ must be positive and this demands $\mathrm{V}_{\mathrm{t}}>\mathrm{V}_{\mathrm{A}}$ which is not possible. In fact, $\mathrm{V}_{\mathrm{GS} 2}=\mathrm{V}_{\mathrm{A}}$ must be greater than $\mathrm{V}_{\mathrm{t}}$.
Thus, $V_{D D}=V_{A}+V_{B}$

Q9. The diodes $D_{l}$ and $D_{3}$ turn on when $v_{\text {in }}>1.2 \mathrm{~V}$ while the diodes $D_{2}$ and $D_{4}$ turn on when $v_{i n}<-1.2 \mathrm{~V}$.
During time $t \in[25 \mathrm{~ms}, 100 \mathrm{~ms}]$, when $\mathrm{V}_{\text {in }}$ decreases from +1.2 V to -1.2 V , all diodes $\mathrm{D}_{1}$, $\mathrm{D}_{2}, \mathrm{D}_{3}$ and $\mathrm{D}_{4}$ remain off and $\mathrm{i}_{\mathrm{L}}=0$.
The equation of line segment $A B$ is
$\mathrm{y}-5=\frac{-10-5}{100-25}(\mathrm{x}-25) ; y$ represents $v_{\text {in }}$ and $x$ represents time $t$, in ms

$$
\mathrm{v}_{\mathrm{in}}=-0.2 \mathrm{t}+10
$$

$1.2=-0.2 \mathrm{t}_{1}+10$ gives $\mathrm{t}_{1}=44 \mathrm{~ms}$ and $-1.2=-0.2 \mathrm{t}_{2}+10$ gives $\mathrm{t}_{2}=56 \mathrm{~ms}$

$$
\Delta \mathrm{t}=\mathrm{t}_{2}-\mathrm{t}_{1}=56-44=12 \mathrm{~ms}
$$

Q10. $I_{\text {ios }}=0 \Rightarrow I_{B 1}-I_{B 2}=0 \Rightarrow I_{B 1}=I_{B 2}$

$$
\begin{aligned}
& \mathrm{V}=-\mathrm{I}_{\mathrm{B} 2} \mathrm{R}_{\mathrm{X}} \text { and } \mathrm{I}_{1}=\frac{0-\mathrm{V}}{\mathrm{R}_{1}}=\frac{\mathrm{I}_{\mathrm{B} 2} \mathrm{R}_{\mathrm{X}}}{\mathrm{R}_{1}} \\
& \mathrm{I}_{2}=\mathrm{I}_{1}-\mathrm{I}_{\mathrm{B} 1}=\frac{\mathrm{I}_{\mathrm{B} 2} \mathrm{R}_{\mathrm{X}}}{\mathrm{R}_{1}}-\mathrm{I}_{\mathrm{B} 1}
\end{aligned}
$$

$$
\mathrm{V}-\mathrm{I}_{2} \mathrm{R}_{2}=\mathrm{V}_{2}=0 \Rightarrow-\mathrm{I}_{\mathrm{B} 2} \mathrm{R}_{\mathrm{X}}-\left[\frac{\mathrm{I}_{\mathrm{B} 2} \mathrm{R}_{\mathrm{X}}}{\mathrm{R}_{1}}-\mathrm{I}_{\mathrm{B} 1}\right] \mathrm{R}_{2}=0 ; \mathrm{I}_{\mathrm{B} 1}=\mathrm{I}_{\mathrm{B} 2}
$$



$$
\mathrm{R}_{\mathrm{X}}+\frac{\mathrm{R}_{\mathrm{X}} \mathrm{R}_{2}}{\mathrm{R}_{1}}=\mathrm{R}_{2} \Rightarrow \mathrm{R}_{\mathrm{X}}=\frac{\mathrm{R}_{1} \mathrm{R}_{2}}{\mathrm{R}_{1}+\mathrm{R}_{2}}=\frac{5 \times 7.5}{5+7.5}=3 k \Omega
$$

Q11(a). For $v_{1}>0$, diode D is reverse biased and $v_{2}=v_{1}$.
For $v_{1}<0$, diode D is forward biased and $v_{2}=-v_{1}$.
Q12(b). $\mathrm{V}_{\mathrm{E} 2}=5-0.7-0.7=3.6 \mathrm{~V}$

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{Y}}=\frac{3.6}{100} \mathrm{~A}=36 \mathrm{~mA} \\
& \mathrm{I}=\mathrm{I}_{\mathrm{C} 1}+\mathrm{I}_{\mathrm{C} 2}=\mathrm{I}_{\mathrm{X}}+\mathrm{I}_{\mathrm{E} 2}=\mathrm{I}_{\mathrm{Y}}=36 \mathrm{~mA}
\end{aligned}
$$



Q13( c). For PLL in lock mode
$\omega_{\mathrm{i}}=\omega_{0}+\mathrm{k}_{\mathrm{v}} \times \mathrm{V}_{\mathrm{C}}$
and $\mathrm{V}_{\mathrm{C}}=\frac{\omega_{\mathrm{i}}-\omega_{0}}{\mathrm{k}_{\mathrm{v}}}=\frac{2 \pi\left(\mathrm{f}_{\mathrm{i}}-\mathrm{f}_{0}\right)}{2 \pi \times 10^{3}}=\frac{250-500}{10^{3}}=-0.25 \mathrm{~V}$

Q14. The circuit for $b_{1} b_{2} b_{3}=011$, is sketched below.

$$
\begin{aligned}
& \mathrm{I}_{0}=\mathrm{I}_{1}+\mathrm{I}_{2} \\
& \frac{0-\mathrm{v}_{0}}{\mathrm{R}}=\frac{5-0}{4 \mathrm{R}}+\frac{5-0}{8 \mathrm{R}} \\
& \mathrm{v}_{0}=-(1.25+0.625) \\
& \quad=-1.875 \mathrm{~V}
\end{aligned}
$$



Q15(b). $\mathrm{I}_{\mathrm{Z}}=\mathrm{I}-\mathrm{I}_{\mathrm{L}}>0$ or $\mathrm{I}>\mathrm{I}_{\mathrm{L}}$ ensures consistent regulation.

$$
\begin{aligned}
& \frac{\mathrm{V}_{\mathrm{S}}-\mathrm{V}_{\mathrm{Z}}}{\mathrm{R}}>\frac{\mathrm{V}_{\mathrm{Z}}}{\mathrm{R}_{\mathrm{L}}} \\
& \mathrm{R}_{\mathrm{L}}>\mathrm{R}\left[\frac{\mathrm{~V}_{\mathrm{Z}}}{\mathrm{~V}_{\mathrm{S}}-\mathrm{V}_{\mathrm{Z}}}\right] \text { or } \mathrm{R}_{\mathrm{L}}>\mathrm{R}\left[\frac{\mathrm{~V}_{\mathrm{S}}}{\mathrm{~V}_{\mathrm{Z}}}-1\right]^{-1}
\end{aligned}
$$



Q16. The diode ac resistance, $\mathrm{r}_{\mathrm{f}}=\frac{\mathrm{V}_{\mathrm{T}}}{\mathrm{I}}=\frac{25 \mathrm{mV}}{0.1 \mathrm{~mA}}=250 \Omega$

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{y}}=\frac{\mathrm{v}_{\mathrm{x}} \mathrm{r}_{\mathrm{f}}}{\mathrm{r}_{\mathrm{f}}+\mathrm{r}_{\mathrm{x}}}=\frac{250 \mathrm{v}_{\mathrm{x}}}{250+250}=\frac{\mathrm{v}_{\mathrm{x}}}{2} \\
& \mathrm{v}_{\mathrm{y}}=0.5 \mathrm{v}_{\mathrm{x}} \Rightarrow \lambda=0.5
\end{aligned}
$$

Q17(b). $\quad \mathrm{v}_{0}(\mathrm{t})=\left[\lambda \cos \left(10^{4} \mathrm{t}\right)\right]\left(1+\frac{90}{10}\right)=10 \lambda \cos 10^{4} \mathrm{t} V$
$\dot{\mathrm{v}}_{0}(\mathrm{t})=10^{5} \lambda \cos 10^{4} \mathrm{t}$ and $\left.\dot{\mathrm{v}}_{0}(\mathrm{t})\right|_{\max }=10^{5} \lambda \mathrm{~V} / \mathrm{sec}$
For no distortion, $\left.\dot{\mathrm{v}}_{0}(\mathrm{t})\right|_{\max } \leq 0.4 \times 10^{6} \Rightarrow 10^{5} \lambda \leq 0.4 \times 10^{6}$
$\lambda \leq 4 \mathrm{~V}$

Q18(b). $\mathrm{v}_{\mathrm{y}}=\mathrm{v}_{\mathrm{x}}$
The current through $\mathrm{R}_{\mathrm{x}}$ is zero.

accelerating pace of learning

Q20. The gain-band width product, $G B P=A_{0} \mathrm{f}_{\mathrm{p}}=(100 \mathrm{~dB}) \times 4 \mathrm{~Hz}=\left(10^{5} \mathrm{~V} / \mathrm{V}\right) \times 4 \mathrm{~Hz}=400 \mathrm{kHz}$
$\left|\mathrm{A}_{\mathrm{pb}}\right|=100$ and band width $\mathrm{f}_{\mathrm{b}}=\frac{\mathrm{A}_{0} \mathrm{f}_{\mathrm{p}}}{\left|\mathrm{A}_{\mathrm{fb}}\right|}=\frac{400 \mathrm{kHz}}{100}$

$$
=4 k H z
$$

It is important to note that GBP remains constant .


Q21(a) The dc equivalent circuit is sketched below with ac source $\mathrm{v}_{\mathrm{s}}$ shorted .Transistors $\mathrm{Q}_{\mathrm{a}}, \mathrm{Q}_{\mathrm{b}}$ and $\mathrm{Q}_{\mathrm{c}}$ are in saturation, have equal drain currents and equal $\mathrm{V}_{\mathrm{GS}}=5 \mathrm{~V}$.
Assume $\mathrm{Q}_{3}$ in saturation.
$\mathrm{I}_{\mathrm{D} 3}=\mathrm{K}_{3}\left(\mathrm{~V}_{\mathrm{GS} 3}-\mathrm{V}_{\mathrm{t}}\right)^{2}=\left[8 \mathrm{~mA} / \mathrm{V}^{2}\right](5-3-1)^{2}=8 \mathrm{~mA}$
$\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ are identical and allow equal drain currents.
Let $V_{G S 1}=V_{G S 2}=V_{1}$

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{D} 1}=\mathrm{I}_{\mathrm{D} 3} \Rightarrow \mathrm{~K}_{1}\left(\mathrm{~V}_{\mathrm{GS} 1}-\mathrm{V}_{\mathrm{t}}\right)^{2}=8 \mathrm{~mA} \Rightarrow\left(\mathrm{~V}_{1}-1\right)^{2}=8 \\
& \mathrm{~V}_{1}=1 \pm 2 \sqrt{2} \mathrm{~V}
\end{aligned}
$$

Since, $\mathrm{V}_{1}$ can not be negative, $\mathrm{V}_{1}=1+2 \sqrt{2}=3.83 \mathrm{~V}$
$V_{\text {OUT }}=15-2 \times 3.83=7.34 \mathrm{~V}$
Note that $\mathrm{V}_{\mathrm{GD} 3}=5-7.34=-2.34<\mathrm{V}_{\mathrm{t}} ; \mathrm{V}_{\mathrm{t}}=1 \mathrm{~V}$,
$Q_{3}$ is, indeed, in saturation.


Q22(a). $\mathrm{I}_{\mathrm{D}_{1}}=\mathrm{I}_{\mathrm{ref}}=\frac{1}{2} \mathrm{~K}_{\mathrm{n}}^{\prime}\left(\mathrm{V}_{\mathrm{GS} 1}-\mathrm{V}_{\mathrm{t}}\right)^{2}$

$$
100 \times 10^{-6}=\frac{1}{2} \times 200 \times 10^{-6}\left(\mathrm{~V}_{\mathrm{GS1}}-1\right)^{2} \Rightarrow \mathrm{~V}_{\mathrm{GSI}}=1 \pm 1=0 \text { or } 2 \mathrm{~V}
$$

Note that $\mathrm{V}_{\mathrm{GS}}$ can not be 0. So, $\mathrm{V}_{\mathrm{GSI}}=2 \mathrm{~V}$ and $\mathrm{R}_{\mathrm{x}}=\frac{5-2}{100 \mu \mathrm{~A}}=30 \mathrm{k} \Omega$
The transistor $\mathrm{Q}_{2}$ will remain in saturation if $\mathrm{V}_{\mathrm{DS}} \geq \mathrm{V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{t}}$ or $\mathrm{V}_{0} \geq 2-1$ or $\mathrm{V}_{0} \geq 1 \mathrm{~V}$. $\mathrm{V}_{0, \text { min }}=1 \mathrm{~V}$

Q23(b).


Q24(d). Corrigendum ; please note that Fig.(b) shows relation between $\mathrm{v}_{\mathrm{x}}$ and $\mathrm{v}_{\mathrm{y}}$, and not between $\mathrm{v}_{\mathrm{x}}$ and $\mathrm{v}_{\mathrm{z}}$. The error in Q24 is regretted.

$$
\begin{aligned}
& \mathrm{v}_{\mathrm{y}}=-\mathrm{v}_{\mathrm{x}} \text { for } \mathrm{v}_{\mathrm{x}}>0 \\
& =0 \text { for } \mathrm{v}_{\mathrm{x}}<0 \\
& \mathrm{v}_{\mathrm{z}}=-\mathrm{v}_{\mathrm{x}}-2 \mathrm{v}_{\mathrm{y}}=-\mathrm{v}_{\mathrm{x}}-2\left(-\mathrm{v}_{\mathrm{x}}\right)=\mathrm{v}_{\mathrm{x}} \quad \text { for } \mathrm{v}_{\mathrm{x}}>0 \\
& =-\left(-\mathrm{v}_{\mathrm{x}}\right)-2 \times 0=\mathrm{v}_{\mathrm{x}} \text { for } \mathrm{v}_{\mathrm{x}}<0
\end{aligned}
$$

The block diagram realizes absolute value operation .

$$
\begin{gathered}
\text { Q25(b). } \mathrm{g}_{\mathrm{m}}=\frac{\mathrm{I}_{\mathrm{C} \mathrm{Q}}}{\mathrm{~V}_{\mathrm{T}}} \Rightarrow \frac{0.8 \mathrm{~mA}}{26 \mathrm{mV}} \leq \mathrm{g}_{\mathrm{m}} \leq \frac{1.2 \mathrm{~mA}}{26 \mathrm{mV}} \text { or } 30.77 \leq \mathrm{g}_{\mathrm{m}} \leq 46.15 \mathrm{~mA} / \mathrm{V} \\
\mathrm{r}_{\pi, \max }=\frac{\beta_{\max }}{\mathrm{g}_{\mathrm{m}, \min }}=\frac{180}{30.77 \times 10^{-3}}=5.85 \mathrm{k} \Omega
\end{gathered}
$$

## Analog circuit Drill

Q1(c). Corrigendum : the value of $v_{-}$at $t=100 \mathrm{~ms}$ is to be evaluated.
At $t=0, \mathrm{v}_{\text {out }}=0$ and $\mathrm{v}_{\mathrm{C}}=0$.
$\mathrm{v}_{\text {out }}=-\frac{\text { In linear region for } \mathrm{t}>0}{20 \times 10^{3} \times 1 \times 10^{-6}} \int_{0}^{\mathrm{t}} \mathrm{v}_{\text {in }}(\mathrm{t}) \mathrm{dt}=-50 \int_{0}^{\mathrm{t}} 2 . \mathrm{dt}=-100 \mathrm{t}$ while $\mathrm{v}_{\mathrm{C}}(\mathrm{t})=0-\mathrm{v}_{\text {out }}=100 \mathrm{t}$.
The output voltage $v_{\text {out }}$ will linearly fall and reach saturation limit of -8 V at time
$\mathrm{t}_{1}=\frac{-8}{-100}=0.08 \mathrm{sec}=80 \mathrm{~ms}$ while $\left.\mathrm{v}_{\mathrm{C}}\right|_{\mathrm{t}=80 \mathrm{~ms}}=8 \mathrm{~V}$
For $t>t_{t}$ or $t>80 \mathrm{~ms}$, the op amp is no more in linear region and inverting terminal of op amp is no more at virtual ground. The capacitor charges exponentially with time constant $\tau=20 \times 10^{3} \times 10^{-6}=20 \times 10^{-3}=20 \mathrm{~ms}$ till the voltage at inverting terminal $v_{-}=2 V$ and $v_{C}=v_{-}-v_{\text {out }}=2-(-8)=10 \mathrm{~V}$.

$$
\begin{aligned}
& \text { For } t>t_{l}, \mathrm{v}_{\mathrm{C}}(\mathrm{t})=\mathrm{v}_{\mathrm{C}}\left(\mathrm{t}_{1}\right)+\left[\mathrm{v}_{\mathrm{C}}(\infty)-\mathrm{v}_{\mathrm{C}}\left(\mathrm{t}_{1}\right)\right]\left(1-\mathrm{e}^{-(t-1) / /}\right)=8+(10-8)\left(1-\mathrm{e}^{-(t-1) / \tau}\right)=8+2\left(1-\mathrm{e}^{-(-1-1) / \tau}\right) \\
& \text { and } \mathrm{v}_{-}=\mathrm{v}_{\text {out }}+\mathrm{v}_{\mathrm{C}}=-8+8+2\left(1-\mathrm{e}^{-(t-1) / \tau}\right)=2\left(1-\mathrm{e}^{-(t-1) / \tau}\right) \\
& \left.\mathrm{v}_{\mathrm{C}}(\mathrm{t})\right|_{\mathrm{t}=100 \mathrm{~ms}}=8+2\left[1-\mathrm{e}^{-(100-80) / 20}\right]=9.26 \mathrm{~V} \\
& \text { At } t=100 m s, \mathrm{v}_{\mathrm{C}}=9.26 \mathrm{~V}, \mathrm{v}_{\text {out }}=-8 \mathrm{~V} \text { and } \mathrm{v}_{-}=\mathrm{v}_{\text {out }}+\mathrm{v}_{\mathrm{C}}=-8+9.26=1.26 \mathrm{~V}
\end{aligned}
$$



$$
\mathrm{I}_{\mathrm{B}_{1}}=\mathrm{I}_{\mathrm{B}_{2}}=0 \text { and } \mathrm{I}_{\mathrm{C} 1}=\frac{5.4-0.7}{4.7 \mathrm{k} \Omega}=1 \mathrm{~mA}
$$

Q2. $\mathrm{I}_{\mathrm{C} 2}=2 \mathrm{I}_{\mathrm{C} 1}=2 \mathrm{~mA}=\mathrm{I}_{\mathrm{C} 3}$ and $\mathrm{g}_{\mathrm{m} 3}=\frac{\mathrm{I}_{\mathrm{C} 3}}{\mathrm{~V}_{\mathrm{T}}}=\frac{2 \mathrm{~mA}}{25 \mathrm{~mA}}=0.08 \mathrm{~A} / \mathrm{V}$
$\mathrm{Q}_{3}$ has $\mathrm{g}_{\mathrm{m}}=0.08 \mathrm{~A} / \mathrm{V}$ and $\mathrm{r}_{\pi}=\frac{\beta}{\mathrm{g}_{\mathrm{m}}}=\infty$

$\mathrm{v}_{0}(\mathrm{t})=\mathrm{g}_{\mathrm{m}} \mathrm{v}_{\pi} \mathrm{r}_{\mathrm{L}}=\mathrm{g}_{\mathrm{m}} \mathrm{v}_{\mathrm{i}}(\mathrm{t}) \mathrm{r}_{\mathrm{L}}=0.08 \times 10^{-2} \cos (1000 \pi \mathrm{t}) \times 2 \times 10^{3} \Rightarrow V_{0}=1.6 \mathrm{~V}$

## olutions

## Test Drill II

Q3.Both transistors $M_{1}$ and $M_{2}$ have $V_{G D}=0$ and therefore, both $M_{1}$ and $M_{2}$ are obviously in saturation .

$$
\begin{aligned}
& \mathrm{I}=0.5 \mathrm{I}_{1} \Rightarrow \mathrm{I}_{1}=\mathrm{I}_{2} \\
& \mathrm{~K}_{1}\left(\mathrm{~V}_{\mathrm{GS} 1}-\mathrm{V}_{\mathrm{t} 1}\right)^{2}=\mathrm{K}_{2}\left(\mathrm{~V}_{\mathrm{GS} 2}-\mathrm{V}_{\mathrm{t} 2}\right)^{2} ; \mathrm{V}_{\mathrm{GS} 1}=\mathrm{V}_{\mathrm{GS} 2}=\mathrm{V}_{\mathrm{x}} \\
& \mathrm{~K}_{1}\left(\mathrm{~V}_{\mathrm{x}}-\mathrm{V}_{\mathrm{t} 1}\right)^{2}=\mathrm{K}_{2}\left(\mathrm{~V}_{\mathrm{x}}-\mathrm{V}_{\mathrm{t} 2}\right)^{2} \\
& 0.5 \times 10^{-3}\left(\mathrm{~V}_{\mathrm{x}}-1\right)^{2}=2 \times 10^{-3}\left(\mathrm{~V}_{\mathrm{x}}-2\right)^{2} \quad \text { or } \mathrm{V}_{\mathrm{x}}-1= \pm 2\left(\mathrm{~V}_{\mathrm{x}}-2\right)
\end{aligned}
$$

$\mathrm{V}_{\mathrm{x}}-1=2\left(\mathrm{~V}_{\mathrm{x}}-2\right)$ gives $\mathrm{V}_{\mathrm{x}}=3 \mathrm{~V}$ and $\mathrm{V}_{\mathrm{x}}-1=-2\left(\mathrm{~V}_{\mathrm{x}}-2\right)$ gives $\mathrm{V}_{\mathrm{x}}=\frac{5}{3} \mathrm{~V}$
Note that $\mathrm{V}_{\mathrm{GS} 1}=\mathrm{V}_{\mathrm{GS} 2}=\mathrm{V}_{\mathrm{x}}$ must be larger than $\mathrm{V}_{\mathrm{t} 1}$ and $\mathrm{V}_{\mathrm{t} 2}$ both. Thus, $V_{x}=3 \mathrm{~V}$
Q4. The circuit is re-sketched below in more convenient form.
$\frac{1-\mathrm{V}_{1}}{\mathrm{R}}=\frac{\mathrm{V}_{1}}{\mathrm{R}}+\frac{\mathrm{V}_{1}-\mathrm{V}}{\mathrm{R}}$ gives $\mathrm{V}_{1}=\frac{1+\mathrm{V}}{3}$
$\frac{\mathrm{V}_{1}-\mathrm{V}}{\mathrm{R}}=\frac{\mathrm{V}}{\mathrm{R}}$ gives $\mathrm{V}=\frac{\mathrm{V}_{1}}{2}=\frac{1}{2}\left[\frac{1+\mathrm{V}}{3}\right] \Rightarrow \mathrm{V}=0.2$
$\frac{1-V_{2}}{R+\Delta R}=\frac{V_{2}}{R}+\frac{V_{2}-V}{R}$ and $\frac{V_{2}-V}{R}=\frac{V-V_{y}}{R}$

$\frac{1-V_{2}}{\mathrm{R}+\Delta \mathrm{R}}=\frac{\mathrm{V}_{2}}{\mathrm{R}}+\frac{\mathrm{V}_{2}-0.2}{\mathrm{R}}$ and $\mathrm{V}_{2}-0.2=0.2-\mathrm{V}_{\mathrm{y}} \Rightarrow \mathrm{V}_{2}=0.4-\mathrm{V}_{\mathrm{y}}$
$\frac{1-\left(0.4-V_{y}\right)}{R+\Delta R}=\frac{0.4-V_{y}}{R}+\frac{0.4-V_{y}-0.2}{R}$
$\frac{0.6+\mathrm{V}_{\mathrm{y}}}{1+\frac{\Delta \mathrm{R}}{\mathrm{R}}}=0.4+0.4-0.2-2 \mathrm{~V}_{\mathrm{y}} ; \quad \frac{\Delta \mathrm{R}}{\mathrm{R}}=0.06$
$\frac{0.6+\mathrm{V}_{\mathrm{y}}}{1+0.06}=0.6-2 \mathrm{~V}_{\mathrm{y}} \Rightarrow \mathrm{V}_{\mathrm{y}}=0.011538 \mathrm{~V}=11.54 \mathrm{mV}$

Q5(c). $\mathrm{g}_{\mathrm{m}}=\frac{\mathrm{I}_{\mathrm{C}}}{\mathrm{V}_{\mathrm{T}}}=\frac{1 \mathrm{~mA}}{25 \mathrm{mV}}=0.04 \mathrm{~A} / \mathrm{V}, \mathrm{r}_{\pi}=\frac{\beta}{\mathrm{g}_{\mathrm{m}}}=\frac{100}{0.04}=2500 \Omega=2.5 \mathrm{k} \Omega$

$$
\mathrm{r}_{0}=\frac{\mathrm{V}_{\mathrm{A}}}{\mathrm{I}_{\mathrm{C}}}=\frac{100 \mathrm{~V}}{1 \mathrm{~mA}}=100 \mathrm{k} \Omega
$$

ac model

$\mathrm{v}_{2}=-\mathrm{g}_{\mathrm{m}} \mathrm{v}_{\pi} \mathrm{r}_{\mathrm{L}}{ }^{*} ; \mathrm{v}_{\pi}=\mathrm{v}_{1}$
ac equivalent circuit
$\frac{\mathrm{v}_{2}}{\mathrm{v}_{1}}=-0.04 \times 7.4 \times 10^{3}=-296$ and $\left|\frac{\mathrm{v}_{2}}{\mathrm{v}_{1}}\right|=296$

Q6. $\mathrm{I}_{\mathrm{C}}=\mathrm{I}_{\mathrm{S}} \mathrm{e}^{\mathrm{V}_{\mathrm{BE}} / \mathrm{V}_{\mathrm{T}}}$ and $\mathrm{V}_{\mathrm{BE}}=\mathrm{V}_{\mathrm{T}} \ln \left(\mathrm{I}_{\mathrm{C}} / \mathrm{I}_{\mathrm{S}}\right)$

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{BE} 1}=\mathrm{V}_{\mathrm{BE} 2}+\mathrm{I}_{\mathrm{C} 2} \mathrm{R}_{2} \\
& \mathrm{~V}_{\mathrm{T}} \ln \left(\frac{\mathrm{I}_{\mathrm{C} 1}}{\mathrm{I}_{\mathrm{S}}}\right)=\mathrm{V}_{\mathrm{T}} \ln \left(\frac{\mathrm{I}_{\mathrm{C} 2}}{\mathrm{I}_{\mathrm{S}}}\right)+5 \times 10^{-6} \mathrm{R}_{2} \\
& \mathrm{I}_{\mathrm{C} 1}=\frac{5-0.7}{4.3 \mathrm{k} \Omega}=1 \mathrm{~mA}
\end{aligned}
$$

$$
\mathrm{V}_{\mathrm{T}} \ln \left(\frac{\mathrm{I}_{\mathrm{C} 1}}{\mathrm{I}_{\mathrm{C} 2}}\right)=5 \times 10^{-6} \mathrm{R}_{2} \Rightarrow 25 \times 10^{-3} \ln \left(\frac{10^{-3}}{5 \times 10^{-6}}\right)=5 \times 10^{-6} \mathrm{R}_{2}
$$


$\mathrm{R}_{2}=0.02649 \times 10^{6} \Omega=26.5 \mathrm{k} \Omega$

Q7. Assuming saturation

$$
\begin{aligned}
\mathrm{I}_{\mathrm{D}} & =\frac{1}{2} \mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}} \frac{\mathrm{~W}}{\mathrm{~L}}\left(\mathrm{~V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{t}}\right)^{2} \\
& =\frac{1}{2} \times 100 \times 10^{-6} \times 50(1-0.8)^{2} \\
& =100 \times 10^{-6} \mathrm{~A}=100 \mu \mathrm{~A} \\
\mathrm{~V}_{\mathrm{D}} & =5-100 \times 10^{-6} \times 5 \times 10^{3}=4.5 \mathrm{~V} \\
\mathrm{~V}_{\mathrm{GD}} & =1-4.5=-3.5 \mathrm{~V}
\end{aligned}
$$



Since, $\mathrm{V}_{\mathrm{GD}}<\mathrm{V}_{\mathrm{t}}$, the saturation mode of opration is justified.

$$
\mathrm{g}_{\mathrm{m}}=\frac{\partial \mathrm{I}_{\mathrm{D}}}{\partial \mathrm{~V}_{\mathrm{GS}}}=\mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}} \frac{\mathrm{~W}}{\mathrm{~L}}\left(\mathrm{~V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{t}}\right)=100 \times 10^{-6} \times 50(1-0.8)=10^{-3} \mathrm{~A} / \mathrm{V}
$$

For $\lambda=0, \mathrm{~V}_{\mathrm{A}}=\infty$ and $\mathrm{r}_{0}=\infty$


$$
\begin{aligned}
& \mathrm{v}_{0}=-\mathrm{g}_{\mathrm{m}} \mathrm{v}_{\mathrm{i}} \times 5 \times 10^{3} ; \mathrm{v}_{\mathrm{i}}=\mathrm{v}_{\mathrm{s}} \\
& \left|\frac{\mathrm{v}_{0}}{\mathrm{v}_{\mathrm{s}}}\right|=10^{-3} \times 5 \times 10^{3}=5
\end{aligned}
$$

## Solutions

Test Drill II

$$
\begin{aligned}
& \text { Q8(c). } \frac{\mathrm{V}_{0}-20}{1 \mathrm{k} \Omega}+\frac{\mathrm{V}_{0}-0}{5 \mathrm{k} \Omega}+\frac{\mathrm{V}_{0}-5}{0.1 \mathrm{k} \Omega}=0 \\
& 5 \mathrm{~V}_{0}-100+\mathrm{V}_{0}+50 \mathrm{~V}_{0}-250=0 \\
& \mathrm{~V}_{\mathrm{D}}
\end{aligned}=6.25 \mathrm{~V} .
$$



Q9. $\mathrm{V}_{\mathrm{C}}(\mathrm{t})=(17.8-1) \mathrm{e}^{-\mathrm{t} / \mathrm{RC}}$

$$
\begin{aligned}
& \dot{\mathrm{V}}_{\mathrm{C}}(\mathrm{t})=\frac{-16.8}{\mathrm{RC}} \mathrm{e}^{-t / \mathrm{RC}} \\
& \dot{\mathrm{v}}_{\mathrm{C}}(0)=\frac{-16.8}{\mathrm{RC}}=\frac{-\mathrm{V}_{\mathrm{R}}}{1 / 60}
\end{aligned}
$$



Peak to peak ripple volttage,$V_{R}=\frac{16.8}{60 \mathrm{RC}}=\frac{16.8}{60 \times 15 \times 25000 \times 10^{-6}}=0.75 \mathrm{~V}$ dc output voltage, $\mathrm{V}_{0}=16.8-\frac{1}{2} \times 0.75=16.4 \mathrm{~V}$
Q10. Until some value of $V_{i}$, both diodes $D_{1}$ and $D_{2}$ are on. Obviously $I_{2}=\left(I_{1}-I\right)>0$ and $V_{0}=4.4 V$
$\mathrm{I}<\mathrm{I}_{1} \Rightarrow \frac{10-4.4}{9.5 \mathrm{k} \Omega}<\frac{4.4-0.6-\mathrm{V}_{\mathrm{i}}}{0.5 \mathrm{k} \Omega} \Rightarrow \mathrm{V}_{\mathrm{i}}<3.5 \mathrm{~V}$
Thus, $\mathrm{V}_{1}=3.5 \mathrm{~V}$
For $\mathrm{V}_{\mathrm{i}}>3.5 \mathrm{~V}, \mathrm{D}_{2}$ turns off

$$
\text { and } \frac{\mathrm{V}_{0}-10}{9.5 \mathrm{k} \Omega}+\frac{\mathrm{V}_{0}-0.6-\mathrm{V}_{\mathrm{i}}}{0.5 \mathrm{k} \Omega}=0 \Rightarrow \mathrm{~V}_{0}=0.95 \mathrm{~V}_{\mathrm{i}}+1.07
$$

Thus, for $\mathrm{V}_{\mathrm{i}}>3.5 \mathrm{~V}, \mathrm{~V}_{0}$ linearly increases as $\mathrm{V}_{\mathrm{i}}$ increases until $\mathrm{V}_{\mathrm{i}}$ equals 9.4 V . Therefore, $\mathrm{V}_{2}=9.4 \mathrm{~V}$
For $\mathrm{V}_{\mathrm{i}}>9.4 \mathrm{~V}, \mathrm{D}_{1}$ also turns off and $\mathrm{V}_{0}=10 \mathrm{~V}$

$\left|\mathrm{V}_{1}-\mathrm{V}_{2}\right|=|3.5-9.4|=5.9 \mathrm{~V}$
Q11(d). Assume that all three diodes $\mathrm{D}_{1}, \mathrm{D}_{2}$ and $\mathrm{D}_{3}$ are on as it appears so .
Thus, the equivalent circuit gives

$$
\frac{5-0}{5 \mathrm{k} \Omega}=\frac{0-0.7-(-10)}{5 \mathrm{k} \Omega}+\mathrm{I}_{2}
$$

or $\mathrm{I}_{2}=-0.86 \mathrm{~mA}$ which is not possible.
Thus, $D_{2}$ is off while $D_{1}$ and $D_{3}$ are on.


## Test Drill II

The circuit is redrawn below with $\mathrm{D}_{2}$ off, to examine the state of diodes $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$.

$$
\begin{aligned}
& \mathrm{I}_{1}=\frac{5-0.7-(-10)}{5 \mathrm{k} \Omega+5 \mathrm{k} \Omega}=1.43 \mathrm{~mA} \\
& \mathrm{I}_{2}=\frac{0-0.7-(-5)}{5 \mathrm{k} \Omega}=0.86 \mathrm{~mA}
\end{aligned}
$$

The direction of both currents $I_{1}$ and $I_{2}$ agree with the direction, the diodes $\mathrm{D}_{1}$ and $\mathrm{D}_{3}$ allow the current to flow .
The diodes $\mathrm{D}_{1}$ and $\mathrm{D}_{3}$ are, in deed, on and
$\mathrm{V}_{1}=5-5 \times 1.43=-2.15 \mathrm{~V}$ and $V_{2}=-0.7$


Q12(d). The circuit is sketched below for $\mathrm{V}_{1}=5 \mathrm{~V}$ and assuming that both diodes $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ are on.
Note that 1.7 V across $1.7 \mathrm{k} \Omega$ resistance gives
$\mathrm{I}=1 \mathrm{~mA}, \mathrm{~V}_{2}=5-1.7=3.3 \mathrm{~V}^{2}$ and $\mathrm{I}_{2}=\frac{3.3-0.7}{4 \mathrm{k} \Omega}=0.65 \mathrm{~mA}$
Then, $\mathrm{I}_{1}=\mathrm{I}-\mathrm{I}_{2}=1-0.65=0.35 \mathrm{~mA}$.
But, the direction of $I_{1}$ is disallowed by diode $D_{1}$. In fact, $D_{1}$ is off.


The circuit with only $\mathrm{D}_{2}$ on, is again sketched below.

$$
\frac{5-\mathrm{V}_{2}}{1.7 \mathrm{k} \Omega}=\frac{\mathrm{V}_{2}-0.7}{4 \mathrm{k} \Omega} \Rightarrow \mathrm{~V}_{2}=3.72 \mathrm{~V}
$$

The current through the diode is $\frac{\mathrm{V}_{2}-0.7}{4 \mathrm{k} \Omega}=0.75 \mathrm{~mA}$, the direction of which agrees with the direction in which the diode allows the current to flow.


For $\mathrm{V}_{2}=10 \mathrm{~V}$ assuming diodes $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ on, the circuit is sketched below. Again 1.7 V across $1.7 \mathrm{k} \Omega$ resistance gives $\mathrm{I}=1 \mathrm{~mA}$
$\mathrm{V}_{2}=10-1.7=8.3 \mathrm{~V}$ and $\mathrm{I}_{2}=\frac{8.3-0.7}{4 \mathrm{k} \Omega}=1.9 \mathrm{~mA}$
$\mathrm{I}_{1}=\mathrm{I}_{2}-\mathrm{I}=1.9-1=0.9 \mathrm{~mA}$, the direction of which agrees with the direction ,the diode $\mathrm{D}_{1}$ allows the current to flow .
Thus, for $\mathrm{V}_{2}=10 \mathrm{~V}$, both diodes $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ are, in deed, on.


## Test Drill II

Q13(c) On varying $V_{1}$, the node voltage $V_{2}$ will remain constant equal to 4.8 V only when the transistor is in saturation and $V_{\mathrm{EC}}=5-4.8=0.2 \mathrm{~V}$.
$\mathrm{I}_{\mathrm{C}, \text { sat }}=\frac{\mathrm{V}_{2}}{4 \mathrm{k} \Omega}=\frac{4.8}{4 \mathrm{k} \Omega}=1.2 \mathrm{~mA}$ and $\mathrm{I}_{\mathrm{B}, \min }=\frac{1.2 \mathrm{~mA}}{80}=0.015 \mathrm{~mA}$
Base current, $\mathrm{I}_{\mathrm{B}} \frac{5-0.8-\mathrm{V}_{1}}{180 \mathrm{k} \Omega} \mathrm{A}=\frac{4.2-\mathrm{V}_{1}}{180} \mathrm{~mA}$
For the transistor to be in saturation, $\mathrm{I}_{\mathrm{B}} \geq \mathrm{I}_{\text {Bmin }}$, that is, $\frac{4.2-\mathrm{V}_{1}}{180} \geq 0.015 \Rightarrow \mathrm{~V}_{1} \leq 1.5 \mathrm{~V}$
Thus $\mathrm{V}_{2}=4.8 \mathrm{~V}$ for $0 \leq V_{1} \leq 1.5 \mathrm{~V}$

Q14. Assume transistor in active mode of operation.
$\beta=\infty \Rightarrow I_{B}=0$
$\frac{5-\mathrm{V}_{\mathrm{B}}}{500 \mathrm{k} \Omega}+\frac{3-\mathrm{V}_{\mathrm{B}}}{500 \mathrm{k} \Omega}+\frac{-5-\mathrm{V}_{\mathrm{B}}}{44 \mathrm{k} \Omega}=0$
gives $\mathrm{V}_{\mathrm{B}}=-3.65 \mathrm{~V}$ and $\mathrm{V}_{\mathrm{E}}=-3.65-0.65=4.3 \mathrm{~V}$
$\mathrm{I}_{\mathrm{E}}=\mathrm{I}_{\mathrm{C}}=\frac{-4.3-(-5)}{5 \mathrm{k} \Omega}=0.14 \mathrm{~mA}$
$\mathrm{V}_{\mathrm{C}}=15-(0.14 \times 50)=8 \mathrm{~V}$
$\mathrm{V}_{\mathrm{CE}}=\mathrm{V}_{\mathrm{C}}-\mathrm{V}_{\mathrm{E}}=8-(-4.3)=12.3 \mathrm{~V}$
and power dissipation $=\mathrm{V}_{\mathrm{CE}} \times \mathrm{I}_{\mathrm{C}}=12.3 \times 0.14=1.7 \mathrm{~mW}$
Since, $\mathrm{V}_{\mathrm{CE}}>\mathrm{V}_{\mathrm{CE}, \text { sat }}$, the transistor is ,indeed, in active mode of operation.


Q15(c). The circuit with transform impedances, is sketched below .

$\frac{\mathrm{X}(\mathrm{s})-0}{10^{3}+\frac{1}{10^{-6} \mathrm{~s}}}=\frac{0-\mathrm{V}(\mathrm{s})}{\frac{1}{10^{-6} \mathrm{~s}}} \Rightarrow \mathrm{~V}(\mathrm{~s})=\frac{-\mathrm{X}(\mathrm{s})}{10^{-3} \mathrm{~s}+1}$
$\frac{\mathrm{V}(\mathrm{s})-0}{10^{4}+\frac{1}{10^{-7} \mathrm{~s}}}=\frac{0-\mathrm{Y}(\mathrm{s})}{10^{5}+\frac{1}{10^{-7} \mathrm{~s}}} \Rightarrow \mathrm{~V}(\mathrm{~s})=\frac{-\mathrm{Y}(\mathrm{s})\left(10^{-3} \mathrm{~s}+1\right)}{10^{-2} \mathrm{~s}+1}$
Now , $\frac{-\mathrm{X}(\mathrm{s})}{10^{-3} \mathrm{~s}+1}=\frac{-\mathrm{Y}(\mathrm{s})\left(10^{-3} \mathrm{~s}+1\right)}{\left(10^{-2} \mathrm{~s}+1\right)} \Rightarrow \mathrm{H}(\mathrm{s})=\frac{\mathrm{Y}(\mathrm{s})}{\mathrm{X}(\mathrm{s})}=\frac{10^{-2} \mathrm{~s}+1}{\left(10^{-3} \mathrm{~s}+1\right)^{2}}=\frac{\mathrm{s}+10^{2}}{\left(\mathrm{~s}+10^{3}\right)^{2}}$
and $H(j \omega)=\frac{10^{-2}(\mathrm{j} \omega)+1}{\left[10^{-3}(\mathrm{j} \omega)+1\right]^{2}}$
The Bode dB plot and pole-zero plot, both are depicted below.



It is obvious that the component of frequency $10^{3} \mathrm{rad} / \mathrm{sec}$, is the most amplified with the gain of 20 dB in the output. The components of $100 \mathrm{rad} / \mathrm{sec}$ and $10^{4} \mathrm{rad} / \mathrm{sec}$, are treated with 0 dB (unity)gain .

Q16.The small signal models are sketched below.

$\mathrm{v}_{0}=-\mathrm{g}_{\mathrm{m} 1} \mathrm{v}_{\mathrm{i}}\left(\mathrm{r}_{01} \| \mathrm{r}_{02}\right)$
$\left|\frac{\mathrm{v}_{0}}{\mathrm{v}_{\mathrm{i}}}\right|=\mathrm{g}_{\mathrm{m} 1}\left(\mathrm{r}_{01}| | \mathrm{r}_{02}\right)$
$\mathrm{g}_{\mathrm{m} 1}=\frac{\partial \mathrm{I}_{\mathrm{D} 1}}{\partial \mathrm{~V}_{\mathrm{GS} 1}}=\mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}} \frac{\mathrm{W}}{\mathrm{L}}\left(\mathrm{V}_{\mathrm{GS} 1}-\mathrm{V}_{\mathrm{tn}}\right)=2 \times 10^{-3}(4-2)=4 \mathrm{~mA} / \mathrm{V}$

$\mathrm{r}_{01}=\left(\lambda_{1} \mathrm{I}_{\mathrm{D} 1}\right)^{-1}$ where $\mathrm{I}_{\mathrm{D} 1}=\frac{1}{2} \mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}} \frac{\mathrm{W}}{\mathrm{L}}\left(\mathrm{V}_{\mathrm{GSI}}-\mathrm{V}_{\mathrm{tn}}\right)^{2}=\frac{1}{2} \times 2 \times 10^{-3}(4-2)^{2}=4 \mathrm{~mA}$
$\mathrm{r}_{01}=(0.01 \times 4 \mathrm{~mA})^{-1}=25 \mathrm{k} \Omega$
$\mathrm{r}_{02}=\left(\lambda_{2} \mathrm{I}_{\mathrm{D} 2}\right)^{-1}=\left(\lambda_{2} \mathrm{I}_{\mathrm{D} 1}\right)^{-1}=(0.025 \times 4 \mathrm{~mA})=10 \mathrm{k} \Omega$
$\left|\frac{\mathrm{v}_{0}}{\mathrm{v}_{\mathrm{i}}}\right|=4 \times 10^{-3} \times \frac{25 \times 10}{25 \times 10} \times 10^{3}=28.6$

## olutions

## Test Drill II

Q17(b).Assume Q1 in saturation mode.

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{D} 1}= \frac{1}{2} \mu_{\mathrm{e}} \mathrm{C}_{\mathrm{ox}}\left(\frac{\mathrm{~W}}{\mathrm{~L}}\right)_{1}\left(\mathrm{~V}_{\mathrm{GS} 1}-\mathrm{V}_{\mathrm{t} 1}\right)^{2} ; \mathrm{V}_{\mathrm{GS} 1}=5 \mathrm{~V} \\
&= \frac{1}{2} \times 3.2 \times 10^{-3}(5-3)^{2}=6.4 \mathrm{~mA} \\
& \mathrm{I}_{\mathrm{D} 2}=\mathrm{I}_{\mathrm{D} 1}=\frac{1}{2} \mu_{\mathrm{e}} \mathrm{C}_{\mathrm{ox}}\left(\frac{\mathrm{~W}}{\mathrm{~L}}\right)_{2}\left(\mathrm{~V}_{\mathrm{GS} 2}-\mathrm{V}_{\mathrm{t} 2}\right)^{2} \\
& \quad 6.4 \times 10^{-3}=\frac{1}{2} \times 0.4 \times 10^{-3}\left(\mathrm{~V}_{\mathrm{GS} 2}-3\right)^{2} \\
&\left(\mathrm{~V}_{\mathrm{GS} 2}-3\right)= \pm 5.66 \Rightarrow \mathrm{~V}_{\mathrm{GS} 2}=8.66 \mathrm{~V} \text { or }-2.66 \mathrm{~V} \\
& \mathrm{~V}_{\mathrm{GS} 2} \text { can not be }-2.66 \mathrm{~V} . \text { Thus, } \mathrm{V}_{\mathrm{GS} 2}=8.66 \mathrm{~V} \\
& \mathrm{~V}_{\mathrm{GS} 2}=15-\mathrm{V}_{\mathrm{OUT}} \Rightarrow \mathrm{~V}_{\mathrm{OUT}}=6.34 \mathrm{~V}=\lambda \\
& \mathrm{V}_{\mathrm{GD} 1}=0-\mathrm{V}_{\mathrm{OUT}}=-6.34 \mathrm{~V}<\mathrm{V}_{\mathrm{t} 1} \text { and therefore }, \mathrm{Q}_{1} \text { is , in deed , in saturation. }
\end{aligned}
$$

dc model


$\frac{\mathrm{v}_{0}}{\mathrm{v}_{\mathrm{s}}}=\frac{-\mathrm{v}_{\mathrm{gs} 2}}{\mathrm{v}_{\mathrm{gs} 1}}=-\frac{\mathrm{g}_{\mathrm{m} 1}}{\mathrm{~g}_{\mathrm{m} 2}} ; \mathrm{i}_{\mathrm{d} 1}=\mathrm{i}_{\mathrm{d} 2} \Rightarrow \mathrm{~g}_{\mathrm{m} 1} \mathrm{v}_{\mathrm{gs} 1}=\mathrm{g}_{\mathrm{m} 2} \mathrm{v}_{\mathrm{gs} 2}$ under no load condtion.
$\mathrm{I}_{\mathrm{D} 1}=\frac{1}{2} \mu_{\mathrm{e}} \mathrm{C}_{\mathrm{ox}}\left(\frac{\mathrm{W}}{\mathrm{L}}\right)_{1}\left(\mathrm{~V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{t} 1}\right)^{2}$ and $\mathrm{g}_{\mathrm{m} 1}=\frac{\partial \mathrm{I}_{\mathrm{D} 1}}{\partial \mathrm{~V}_{\mathrm{GS} 1}}=\mu_{\mathrm{e}} \mathrm{C}_{\mathrm{ox}}\left(\frac{\mathrm{W}}{\mathrm{L}}\right)_{1}\left(\mathrm{~V}_{\mathrm{GS}}-\mathrm{V}_{\mathrm{t} 1}\right)=\sqrt{2 \mathrm{I}_{\mathrm{D} 1} \mu_{\mathrm{e}} \mathrm{C}_{\mathrm{ox}}\left(\frac{\mathrm{W}}{\mathrm{L}}\right)_{1}}$
Similarly,$g_{m 2}=\sqrt{2 \mathrm{I}_{\mathrm{D} 2} \mu_{\mathrm{e}} \mathrm{C}_{\mathrm{OX}}\left(\frac{\mathrm{W}}{\mathrm{L}}\right)_{2}}$
$\frac{\mathrm{v}_{0}}{\mathrm{v}_{\mathrm{s}}}=-\frac{\sqrt{2 \mathrm{I}_{\mathrm{D} 1} \mu_{\mathrm{e}} \mathrm{C}_{\mathrm{OX}}\left(\frac{\mathrm{W}}{\mathrm{L}}\right)_{1}}}{\sqrt{2 \mathrm{I}_{\mathrm{D} 2} \mu_{\mathrm{e}} \mathrm{C}_{\mathrm{OX}}\left(\frac{\mathrm{W}}{\mathrm{L}}\right)_{2}}}=-\sqrt{\left(\frac{3.2}{0.4}\right)}=-2.8$
$\mathrm{v} 0=-2.8 \times 0.5 \sin \omega \mathrm{t}=-1.4 \sin \omega \mathrm{t} \Rightarrow \mu=-1.4$
Thus, $\lambda=6.34$ and $\mu=-1.4$

Q18(a).

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{BQ}}=\frac{5-0.7}{(180+1) \times 10 \mathrm{k} \Omega+2.5 \mathrm{k} \Omega}=2.37 \mu \mathrm{~A} \\
& \mathrm{I}_{\mathrm{CQ}}=\beta \mathrm{I}_{\mathrm{BQ}}=180 \times 2.37 \mu \mathrm{~A}=0.427 \mathrm{~mA} \\
& \mathrm{~g}_{\mathrm{m}}=\frac{\mathrm{I}_{\mathrm{CQ}}}{\mathrm{~V}_{\mathrm{T}}}=\frac{0.427 \mathrm{~mA}}{25 \mathrm{mV}}=0.017 \mathrm{~A} / \mathrm{V} \\
& \mathrm{r}_{\pi}=\frac{\beta}{\mathrm{g}_{\mathrm{m}}}=\frac{180}{0.017}=10.6 \mathrm{k} \Omega
\end{aligned}
$$



$$
\begin{aligned}
& \mathrm{i}_{0}=-\frac{1}{2} \mathrm{~g}_{\mathrm{m}} \mathrm{v}_{\pi}=-0.5 \times 0.017 \times \frac{10.6}{10.6+2.5} \mathrm{v}_{\mathrm{s}}=-6.88 \times 10^{-3} \mathrm{v}_{\mathrm{s}} \\
& \mathrm{i}_{0}=-6.88 \times 10^{-3} \times(4 \sin \omega \mathrm{t}) \times 10^{-3} \mathrm{~A}=-27.52 \sin \omega \mathrm{t} \mu \mathrm{~A} \\
& \lambda=-27.52
\end{aligned}
$$

Q19(b). $\frac{\mathrm{V}_{\mathrm{X}}(\mathrm{s})-\mathrm{V}(\mathrm{s})}{1 \mathrm{k} \Omega}=\frac{\mathrm{V}(\mathrm{s})-\mathrm{V}_{\mathrm{Y}}(\mathrm{s})}{1 \mathrm{k} \Omega}$

$$
\mathrm{V}(\mathrm{~s})=0.5 \mathrm{~V}_{\mathrm{X}}(\mathrm{~s})+0.5 \mathrm{~V}_{\mathrm{Y}}(\mathrm{~s})
$$

$$
\text { and } \frac{\mathrm{V}_{\mathrm{x}}(\mathrm{~s})-\mathrm{V}(\mathrm{~s})}{\mathrm{R}}=\frac{\mathrm{V}(\mathrm{~s})-0}{1 / \mathrm{Cs}}
$$

$$
\mathrm{V}_{\mathrm{x}}(\mathrm{~s})=\mathrm{V}(\mathrm{~s})(1+\mathrm{sRC})
$$

$$
\mathrm{V}_{\mathrm{X}}(\mathrm{~s})=\left[0.5 \mathrm{~V}_{\mathrm{X}}(\mathrm{~s})+0.5 \mathrm{~V}_{\mathrm{Y}}(\mathrm{~s})\right](1+\mathrm{sRC})
$$

$$
\mathrm{V}_{\mathrm{X}}(\mathrm{~s})[1-0.5-0.5 \mathrm{~s} \mathrm{RC}]=0.5(1+\mathrm{s} \mathrm{RC}) \mathrm{V}_{\mathrm{Y}}(\mathrm{~s})
$$


$\frac{V_{Y}(s)}{V_{X}(s)}=H(s)=\frac{1-s R C}{1+s R C}$ and $H(j \omega)=\frac{1-j \omega R C}{1+j \omega R C}$
$|\mathrm{H}(\mathrm{j} \omega)|=1$ and $\angle \mathrm{H}(\mathrm{j} \omega)=-2 \tan ^{-1} \omega \mathrm{RC}$

$$
-2 \tan ^{-1} \omega \mathrm{RC}=-\frac{\pi}{6} \Rightarrow 10^{4} \times \mathrm{R} \times 10 \times 10^{-9}=\tan \left(\frac{\pi}{12}\right) \Rightarrow \mathrm{R}=2.68 \mathrm{k} \Omega
$$

$|H(j \omega)|=-1$ regardless of $\omega$ and therefore,$\lambda=0.1$

## olutions

## Test Drill II

## Q20.

$\mathrm{g}_{\mathrm{m}}=\frac{\mathrm{I}_{\mathrm{C}}}{\mathrm{V}_{\mathrm{T}}}=\frac{1 \mathrm{~mA}}{25 \mathrm{mV}}=0.4 \mathrm{~A} / \mathrm{V}, \alpha=1 \Rightarrow \mathrm{r}_{\pi}=\infty$ and $\mathrm{V}_{\mathrm{A}}=\infty \Rightarrow \mathrm{r}_{0}=\infty$
SW closed implies $\mathrm{R}=0$ and $\mathrm{v}_{0}=-\mathrm{g}_{\mathrm{m}} \mathrm{v}_{\pi} \mathrm{R}_{\mathrm{L}}$
KVL around input loop gives
$v_{s}-v_{\pi}-g_{m} v_{\pi} \times \frac{1}{j \omega C}=0$
$v_{\pi}=\frac{v_{s} \times j \omega C}{g_{m}+j \omega C}$
$v_{0}=\left(-g_{m} R_{L}\right) v_{s}\left[\frac{j \omega C}{g_{m}+j \omega C}\right]$

$\frac{v_{0}}{v_{s}}=\frac{-g_{m} R_{L}}{1-j \frac{g_{m}}{\omega C}} \Rightarrow \omega_{L}=\frac{g_{m}}{C}$ on comparing with general form of gain $\left|A_{v}\right|=\left|\frac{v_{0}}{v_{s}}\right|=\frac{A_{0}}{1-j \frac{\omega_{L}}{\omega}}$
With SW open, again $v_{0}=-g_{m} v_{\pi} R_{L}$ and $v_{s}-v_{\pi}-g_{m} v_{\pi}\left(R+\frac{1}{j \omega C}\right)=0$
$v_{\pi}=\frac{v_{s} \times j \omega C}{g_{m}+j \omega C\left(1+g_{m} R\right)}$ and $v_{0}=-g_{m} R_{L} v_{s}\left[\frac{j \omega C}{g_{m}+j \omega C\left(1+g_{m} R\right)}\right]$
$\left|\frac{v_{0}}{v_{s}}\right|=\frac{g_{m} R_{L}}{\left(1+g_{m} R\right)+\frac{g_{m}}{j \omega C}}=\frac{\frac{g_{m} R_{L}}{1+g_{m} R}}{1-\frac{j}{\omega} \times\left[\frac{g_{m}}{\left(1+g_{m} R\right) C}\right]} \Rightarrow \omega_{L}^{*}=\frac{g_{m}}{\left(1+g_{m} R\right) C}$
$\omega_{\mathrm{L}}^{*}=0.2 \omega_{\mathrm{L}} \Rightarrow \frac{\mathrm{g}_{\mathrm{m}}}{\left(1+\mathrm{g}_{\mathrm{m}} \mathrm{R}\right) \mathrm{C}}=0.2 \frac{\mathrm{~g}_{\mathrm{m}}}{\mathrm{C}} \Rightarrow 1+\mathrm{g}_{\mathrm{m}} \mathrm{R}=5$ and $\mathrm{R}=\frac{4}{\mathrm{~g}_{\mathrm{m}}}=\frac{4}{0.04}=100 \Omega$

Q21(c). Fig.(1) ; Let the input signal to block with gain $\mathrm{A}_{1}=10^{3}$ be $\mathrm{v}_{\mathrm{E}}$.

$$
\begin{aligned}
& \mathrm{v}_{0}^{*}=\mathrm{A}_{1} \mathrm{~A}_{1} \mathrm{v}_{\mathrm{E}}+\mathrm{A}_{2} \mathrm{v}_{\mathrm{ni}}, \mathrm{v}_{\mathrm{E}}=\mathrm{v}_{\mathrm{i}}-\beta \mathrm{v}_{0}^{*} \\
& \mathrm{v}_{0}^{*}=\mathrm{A}_{1} \mathrm{~A}_{2} \mathrm{v}_{\mathrm{i}}-\mathrm{A}_{1} \mathrm{~A}_{2} \beta \mathrm{v}_{0}^{*}+\mathrm{A}_{2} \mathrm{v}_{\mathrm{ni}} \\
& \mathrm{v}_{0}^{*}=\frac{\mathrm{A}_{1} \mathrm{~A}_{2}}{1+\mathrm{A}_{1} \mathrm{~A}_{2} \beta} \mathrm{v}_{\mathrm{i}}+\frac{\mathrm{A}_{2}}{1+\mathrm{A}_{1} \mathrm{~A}_{2} \beta} \mathrm{v}_{\mathrm{ni}}=\frac{10^{4}}{1+10^{2}} \mathrm{v}_{\mathrm{i}}+\frac{10}{1+10^{2}} \mathrm{v}_{\mathrm{ni}} \simeq 100 \mathrm{v}_{\mathrm{i}}+0.1 \mathrm{v}_{\mathrm{ni}} \\
& (\mathrm{SNR})_{\mathrm{o}}=\frac{100}{0.1} \times \frac{\mathrm{v}_{\mathrm{i}}}{\mathrm{v}_{\mathrm{ni}}}=1000(\mathrm{SNR})_{\mathrm{i}}
\end{aligned}
$$

Fig.(2); $\mathrm{v}_{0}^{*}=\mathrm{A}_{1} \mathrm{~A}_{2} \mathrm{v}_{\mathrm{E}}=\mathrm{A}_{1} \mathrm{~A}_{2}\left[\mathrm{v}_{\mathrm{i}}+\mathrm{v}_{\mathrm{ni}}-\beta \mathrm{v}_{0}^{*}\right]$

$$
\begin{aligned}
& \mathrm{v}_{0}^{*}=\frac{\mathrm{A}_{1} \mathrm{~A}_{2}}{1+\mathrm{A}_{1} \mathrm{~A}_{2} \beta} \mathrm{v}_{\mathrm{i}}+\frac{\mathrm{A}_{1} \mathrm{~A}_{2}}{1+\mathrm{A}_{1} \mathrm{~A}_{2} \beta} \mathrm{v}_{\mathrm{ni}}=\frac{10^{4}}{1+10^{4} \times 0.01} \mathrm{v}_{\mathrm{i}}+\frac{10^{4}}{1+10^{4} \times 0.01} \mathrm{v}_{\mathrm{i}} \cong 100 \mathrm{v}_{\mathrm{i}}+100 \mathrm{v}_{\mathrm{ni}} \\
& (\mathrm{SNR})_{o}=\frac{100}{100} \times \frac{\mathrm{v}_{\mathrm{i}}}{\mathrm{v}_{\mathrm{ni}}}=(\mathrm{SNR})_{\mathrm{i}}
\end{aligned}
$$

Amplifier configuration of Fig(1) has better noise immunity.

## Test Drill II

Q22(d). $\mathrm{T}(\mathrm{f})=|\mathrm{T}(\mathrm{f})| \angle \mathrm{T}(\mathrm{f})=\frac{\lambda}{\left(1+10^{-10} \mathrm{f}^{2}\right)^{3 / 2}} \angle-3 \tan ^{-1} 10^{-5} \mathrm{f}$

$$
-3 \tan ^{-1} 10^{-5} \mathrm{f}=-180^{\circ} \text { gives } \mathrm{f}=10^{5} \tan 60^{\circ}=\sqrt{3} \times 10^{5} \mathrm{~Hz}
$$

$$
|\mathrm{T}(\mathrm{f})|_{\mathrm{f}=\sqrt{3} \times 10^{5}}=\frac{\lambda}{\left[1+10^{-10} \times\left(\sqrt{3} \times 10^{5}\right)^{2}\right]^{\frac{3}{2}}}=\frac{\lambda}{8}
$$

For the amplifier to be stable $\frac{\lambda}{8}<1$ or $\lambda<8$.


Q23. For $\mathrm{v}_{0}=0, \mathrm{I}_{\mathrm{D} 3}=2 \mathrm{~mA}=\frac{1}{2} \mu_{\mathrm{n}} \mathrm{C}_{\mathrm{ox}} \frac{\mathrm{W}}{\mathrm{L}}\left(\mathrm{V}_{\mathrm{GS} 3}-\mathrm{V}_{\mathrm{T}}\right)^{2}$
$2 \times 10^{-3}=\frac{1}{2} \times 2 \times 10^{-3}\left(\mathrm{~V}_{\mathrm{GS} 3}-1\right)^{2} \Rightarrow \mathrm{~V}_{\mathrm{GS} 3}=-0.414 \mathrm{~V}$ or 2.414 V
$\mathrm{V}_{\mathrm{GS} 3}$ must be greater than $\mathrm{V}_{\mathrm{T}}=1 \mathrm{~V}$. Thus, $\mathrm{V}_{\mathrm{GS} 3}=2.414 \mathrm{~V}$
$\mathrm{V}_{\mathrm{G} 3}=2.414 \mathrm{~V}$ and current $\mathrm{I}_{\mathrm{D} 2}$ in $\mathrm{M}_{2}$ is 0.5 mA .
$\mathrm{R}=\frac{12-\mathrm{V}_{\mathrm{G} 3}}{0.5 \mathrm{~mA}}=\frac{12-2.414}{0.5 \mathrm{~mA}}=19.2 \mathrm{k} \Omega$
Note that $\mathrm{V}_{\mathrm{GD} 3}=\mathrm{V}_{\mathrm{G} 3}-\mathrm{V}_{\mathrm{D} 3}=2.414-12=-9.586 \mathrm{~V}<\mathrm{V}_{\mathrm{T}}$. $M_{3}$ is , indeed, in saturation.

Q24( c). $\mathrm{V}_{0}=\mathrm{V}_{\mathrm{Z}}\left[1+\frac{32 \mathrm{k} \Omega}{40 \mathrm{k} \Omega}\right]=5.6 \times 1.8 \cong 10 \mathrm{~V}$
Set $\mathrm{I}_{\mathrm{F}}$ for minimum bias current of 1 mA to get

$$
\mathrm{R}_{\mathrm{F}}=\frac{10-5.6}{1 \mathrm{~mA}}=4.4 k \Omega
$$



The max imum Zener current should be no more than 1.25 mA .
Set $\mathrm{I}_{\mathrm{D} 1}=0.25 \mathrm{~mA}$
$\mathrm{I}_{2}=\frac{5.6+0.7}{30 \mathrm{k} \Omega}=0.21 \mathrm{~mA}$
$\mathrm{I}_{1}=\frac{10-6.3}{\mathrm{R}_{\mathrm{S}}}=(0.21+0.25) \mathrm{mA} \Rightarrow \mathrm{R}_{\mathrm{S}} \cong 8 k \Omega$

Q25. $\mathrm{I}_{\mathrm{X}}=\frac{\mathrm{V}_{1}-\mathrm{V}_{2}}{\mathrm{R}} \Rightarrow \frac{0.25-(-0.25)}{\mathrm{R}}=5 \mathrm{~mA} \Rightarrow \mathrm{R}=100 \Omega$

$$
\begin{aligned}
& \frac{\mathrm{V}_{01}-\mathrm{V}_{1}}{\mathrm{R}_{2}}=\mathrm{I}_{\mathrm{x}} \Rightarrow \mathrm{~V}_{01}=\mathrm{V}_{1}+\mathrm{I}_{\mathrm{x}} \mathrm{R}_{2}=0.25+5 \times 10^{-3} \times 1000=5.25 \mathrm{~V} \\
& \mathrm{~V}_{02}=-0.25 \mathrm{~V} \text { and } \mathrm{V}_{01}-\mathrm{V}_{02}=5.25-(-0.25)=5.5 \mathrm{~V}
\end{aligned}
$$

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